Application of the thermal inactivation model to model risk to human health from consumption of VTEC O157 in beef burgers

Summary Report

May 18, 2015

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1 Introduction

In November 2014, the previously developed APHA quantitative risk assessment for estimating the impact on human health from the consumption of VTEC O157 within a beef burger and other food products (Kosmider *et al*., 2010) was updated to consider a recent increase in the consumption of rare burgers. Within the risk assessment, the inactivation of VTEC O157 during cooking is modelled using a relatively simple approach (see Cassin *et al.*, 1998 for details). Specifically, the log reduction of VTEC O157 is modelled as a function of the internal temperature of the burger without consideration of the size of the burger or the variation in temperature within the burger as an external temperature is applied. To address this, a thermodynamic inactivation model developed by the RIVM and previously used within an EFSA funded risk assessment for *Salmonella* in pork products (Anon, 2007) has been used to better refine the characteristics of the cooking process and resulting impact on VTEC O157. More specifically, the RIVM thermodynamic model divides the burger into a number grid cells dependent on its thickness and width and simulates the heat flow through these cells as it is cooked and the resulting inactivation of bacteria within each cell. The outputs from the thermodynamic model include the time to reach a desired internal cooking temperature and the overall inactivation of VTEC O157 within the burger. The latter was then used as an input to the APHA model to provide the number of human infections of VTEC O157 per 100,000 servings. This report provides an overview of the RIVM thermodynamic model, the scenarios considered in agreement with the FSA, and a summary of the results.

2 RIVM thermal inactivation model overview

2.1 The Cassin model

In the original AHPA risk assessment (Kosmider, Nally et al. 2010) as well as in the recently updated risk assessment for the FSA (Berriman, Kosmider et al. 2014) the model of (Cassin, Lammerding et al. 1998) was used to describe the inactivation of VTEC O157 during the cooking process. The inactivation is modelled as a function of the final internal temperature of the burger (*T*). The log reductions obtained are

$$I(T) = -10.165 + 0.211T$$

Here, *I(T)* is the inactivation factor measured in *log(CFU)*[1], and the inactivation may be calculated as

$$N_{\text{after cooking}} = N_{\text{before cooking}} - I(T)$$

where $N_{before\ cooking}$ and $N_{after\ cooking}$ are the numbers of VTEC O157 prior to and after cooking, respectively. However, this approach has several drawbacks:

[1] The authors in (Cassin, Lammerding et al. 1998) state that the unit for *I*(*T*) is log CFU/g, which must be a mistake, since the weight cannot influence the inactivation factor.

1. The inactivation is based on the final temperature only, while inactivation also takes place before the final temperature is reached
2. The time of cooking is also of importance
3. The final temperature is measured at the core of the burger. Temperatures at sites far removed from the core will have a higher temperature
4. Variation in burger sizes is not accounted for
5. Variation in cooking styles is not accounted for (save for the single parameter *T*, final temperature)

In the following subsections we will discuss how the RIVM thermodynamic model improves upon the Cassin model by incorporating the missing features described above. Technical details on the thermodynamic model are presented in Appendix 1, which was taken from (Anonymous 2007a).

2.2 Time-temperature profiles

Within the RIVM thermal inactivation model, both time and temperature are taken into account. This is accomplished in the model framework using D-values and z-values, where a D-value is the time needed for a log reduction at a reference temperature (typically 60 °C) and a z-value is the temperature increase needed for a tenfold increase of the D-value. Specifically, inactivation over t minutes may be written,

$$I(t) = I(0)10^{-\frac{t}{D_T}}$$

$$D_T = D_{60}10^{-\frac{T-60}{z}}$$

The first equation describes the inactivation during heating, and the second equation describes the change in D-value with changing temperature. In contrast to the formulas in Section 2.1, the kinetics is now expressed in CFU, not log(CFU). Previously reported D and z-values are collected, analysed and summarized in (Anon 2007b). Based on this re-analysis of several previous studies, the values D=1.8 [min^{-1}] and z=6.0 [$°C^{-1}$] are assumed for this study.

2.3 Cooking process

In the RIVM thermal inactivation model, cooking of the burger (patty) is modelled assuming the following steps, which are based on the cooking process described by (Bergsma, Fischer et al. 2007):

1. The patty is cooked on one side for one minute at high temperature (150°C). At this time, cooking temperature and ambient temperature are fixed. Heat flows through the sides of the patty depending on the difference in temperature with the environment (ambient temperature).
2. The product is turned. It is assumed that a crust has formed on the cooked side.
3. One minute later, the heat is lowered to 100°C
4. The product is then cooked depending on the assumed cooking style.

3 Scenarios considered

3.1 Weight and size

In the recently updated APHA model it was assumed that during further processing either a pack of 4 or 2 burgers were produced weighing either 113 grams or 85 grams. For the thermal inactivation model, the dimensions of the burgers are required so a small ad-hoc survey conducted by R. Kosmider (APHA) was undertaken and revealed that the dimensions of a 113g burger are 8.5cm (diameter) and 2.5cm (depth). The dimensions of the smaller 85 gram burger could not be obtained so it was assumed, in agreement with FSA, these were 1 cm in depth. In addition, the FSA requested consideration of 'gourmet' style burgers, which in the absence, of data were assumed to be 5 cm high and weigh 227 grams. The characteristics of the burgers are summarised in Table 1.

Table 1: Summary of the characteristics of the burgers considered within the model

Burger	**Height**	**Diameter**	**Weight**
Small	1 cm	8.5 cm	85 g
Standard	2.5 cm	8.5 cm	113 g
Gourmet	5 cm	8.5 cm	227 g

The dimensions are used within the RIVM thermal inactivation model whereas the respective weights are required for the APHA model.

3.2 Internal cooking temperatures

In the original APHA model, the cooking preferences incorporated were well done, medium and rare, with mean internal temperatures of 68.3, 62.7 and 54.4 ^{o}C respectively. These same temperature profiles are assumed within the RIVM thermodynamic model. However, as the model produces varying temperatures over time, we assume the cooking process is stopped immediately when these desired internal temperatures are reached. The time taken to reach the required internal temperature is noted.

FSA guidelines stipulate a cooking temperature of 70 °C for 2 minutes which is considered sufficient for a 6-log reduction (Anon 2007b). Based on preliminary analysis of various input combinations into the RIVM thermodynamic model, it was agreed with the FSA to include a scenario of 'well-done + 2 minutes'. In this scenario, the burger is cooked until an internal temperature of 68.3°C is reached and then it is cooked for a further 2 minutes before the cooking process is stopped. During the additional 2 minutes of cooking, the internal temperature of the burger may exceed 68.3°C.

3.3 Summary scenarios and model implementation

In summary eighteen scenarios were considered namely,

- Scenarios A1 to A6: small burgers cooked rare (A1), medium (A2), well done (A3), well done plus 2 extra minutes cooking (A4), cooking that achieves a 4 log (A5) and 6 log (A6) reduction,
- Scenarios B1 to B6: standard burgers cooked rare (B1), medium (B2), well done (B3), well done plus 2 extra minutes cooking (B4), cooking that achieves a 4 log (B5) and 6 log (B6) reduction,
- Scenarios C1 to C6: gourmet burgers cooked rare (C1), medium (C2), well done (C3) and well done plus 2 extra minutes cooking (C4), cooking that achieves a 4 log (C5) and 6 log (C6) reduction.

The two outputs from the RIVM thermal inactivation model, developed in MATLAB (Mathworks inc.), were (i) the time to reach the internal desired cooking temperature and (ii) the log reduction for each of the scenarios. The latter output was used as an input to the APHA risk assessment model, developed in @Risk (Palisade corp), to represent the reduction in VTEC O157 after cooking. For each scenario, the APHA model was run 1 million iterations to yield a number of human infections per 100,000 servings. In doing so, all other model parameters remained the same as previously described (Berriman et al., 2014). Further, as per the recent update in late 2014, it is assumed that the entire burger is consumed.

4 Results

4.1 RIVM thermal inactivation model

The time required to reach the desired internal temperature and the log reduction observed for each scenario is summarised in Table 2.

Table 2: Summary of the log reduction and time to reach the internal cooking temperature (i.e. rare 54.4°C, medium 62.7°C and well-done 63.3°C) **for the 18 scenarios considered, from the RIVM thermal inactivation model**

Scenario	Burger size	Cooking preference/desired log reduction	Time to reach internal cooking temperature (min)	Log reduction
A1	Small	Rare	0.5	0.2
A2	Small	Medium	0.9	0.3
A3	Small	Well-done	1.1	0.3
A4	Small	Well-done + 2	3.1	Total inactivation
A5	Small	4 log reduction	≈ 2.3	
A6	Small	6 log reduction	≈ 2.3	
B1	Standard	Rare	2.5	0.5
B2	Standard	Medium	4.2	1.1
B3	Standard	Well-done	5.3	4.1
B4	Standard	Well-done + 2	7.3	Total inactivation
B5	Standard	4 log reduction	5.3	
B6	Standard	6 log reduction	5.6	
C1	Gourmet	Rare	10.6	0.8
C2	Gourmet	Medium	22.8	6.5
C3	Gourmet	Well-done	36.7	9
C4	Gourmet	Well-done + 2	38.7	Total inactivation
C5	Gourmet	4 log reduction	20.1	
C6	Gourmet	6 log reduction	22.3	

Scenario's A4-A6, B3-B6 and C2-C6 give considerable log-reductions, while the remaining scenarios give low to moderate reductions. Figures 1, 3 and 5 show the time required for the internal temperature of a burger (of various sizes) to reach a pre-defined temperature, corresponding to a cooking preference (i.e. rare 54.4°C, medium 62.7°C and well-done 63.3°C). Figures 2, 4 and 6 show the log reductions corresponding to these time-temperature profiles. It is clear from Table 2 and Figures 1 to 6 that the thickness of the burger influences the heating of the burger and resulting inactivation significantly.

The time it takes to observe a 4 and 6 log reduction for the small (1cm) and standard (2.5cm) burgers were estimated to align with the scenarios considered within the APHA risk assessment model in November 2014 whereby a 4 and 6 log reduction was modelled for both an 85 g and 113 g burger. In addition to this, the time for a 4 and 6 log reduction for gourmet burgers (5 cm) was estimated. As can be seen from Figure 1, for the small burgers, to achieve both a 4- and 6-log reduction requires approximately 2.3 minutes of cooking. This can be explained by Figure 2, which shows an extremely steep decrease in log inactivation.

For both the 4- and 6-log reductions in small burgers, the internal temperature of the burger is just over 90 °C (Figure 1). For the standard burgers, a 4 log reduction will require 5.3 minutes of cooking and a 6 log reduction requires 5.6 minutes of cooking at just under 70 °C (Figure 3). For the gourmet burgers, a 4 log reduction requires 20.1 minutes of cooking and a 6 log reduction requires 22.3 minutes of cooking (Figure 5).

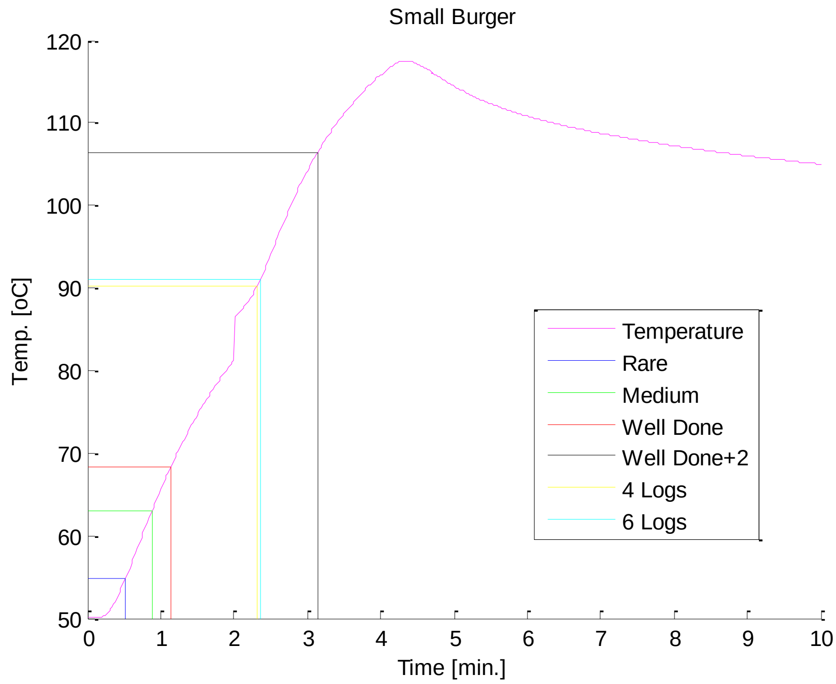

Figure 1: Time-temperature profile for small burgers, i.e. scenarios A1-A6. Note that the line indicates the time required to reach the specified internal temperature.

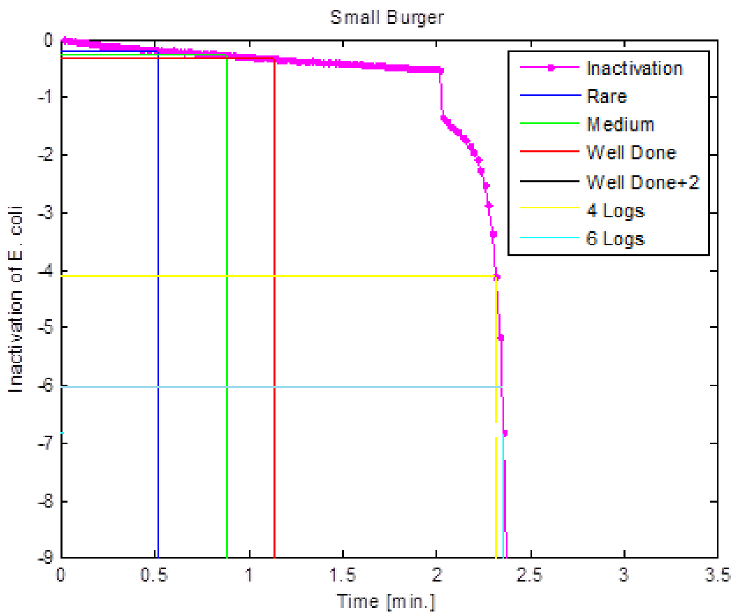

Figure 2: Time inactivation profile for small burgers, i.e. scenarios A1-A6. Note that (i) the well-done plus 2 minutes is not shown as total inactivation occurs (ii) the inactivation of *E. coli* equates to the log reduction at each time point.

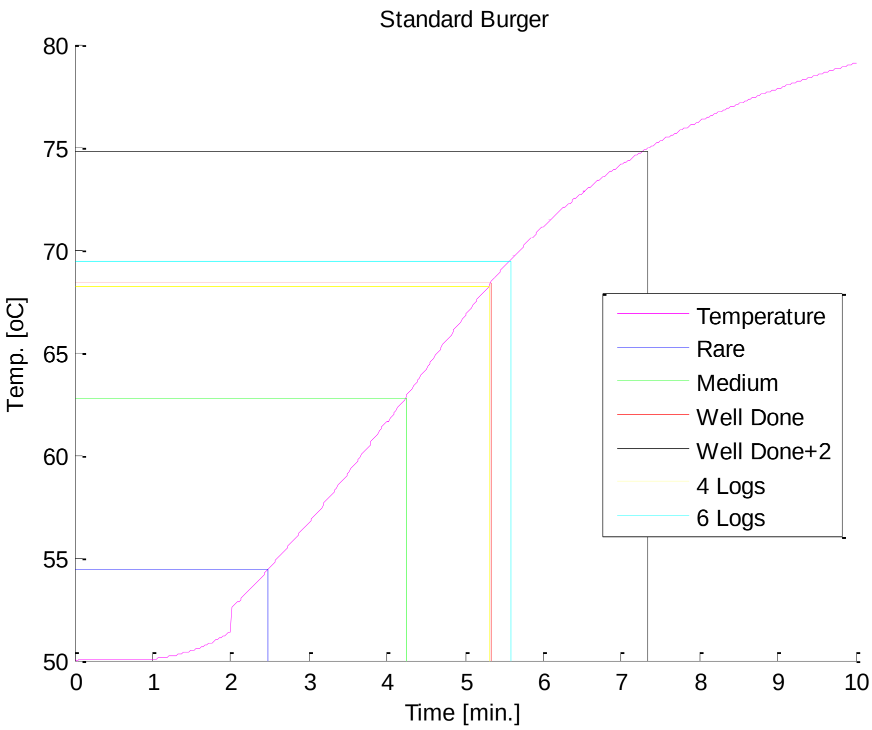

Figure 3: Time-temperature profile for standard burgers, i.e. scenarios B1-B6.

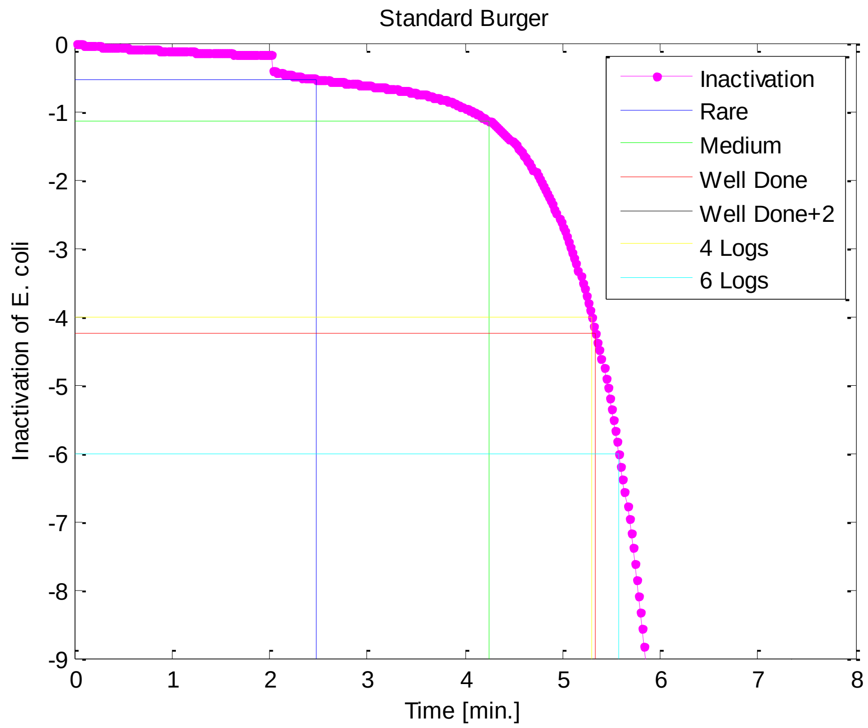

Figure 4: Time inactivation profile for standard burgers, i.e. scenarios B1-B6. Note that well-done plus 2 minutes is not shown as total inactivation occurs.

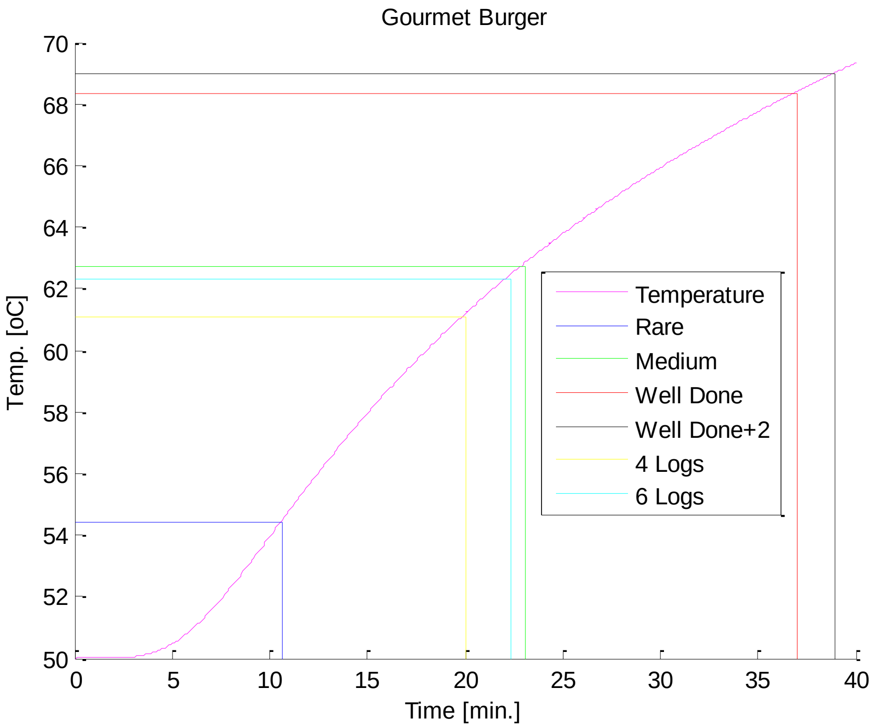

Figure 5: Time-temperature profile for gourmet burgers, i.e. scenarios C1-C6.

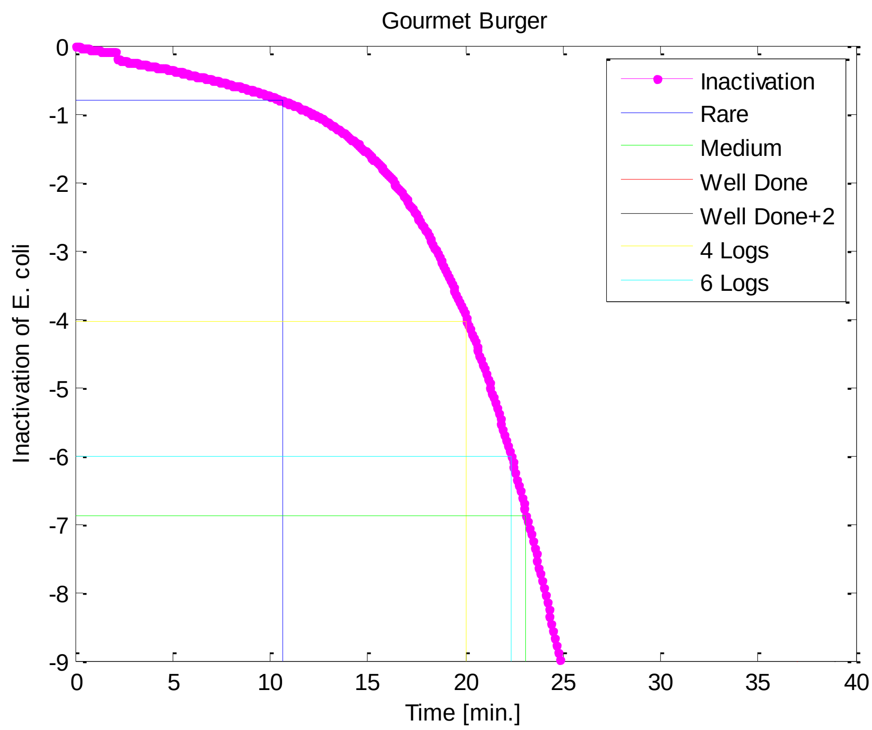

Figure 6: Time inactivation profile for gourmet burgers, i.e. scenarios C1-C6. Note that well-done and well-done plus 2 minutes are not shown as total inactivation occurs.

4.2 APHA risk assessment model

The APHA model was run for 1 million iterations in order to capture the variability/uncertainty contained within the model. As per the original model, the main output is the mean number of infections per 100,000 servings of burgers. A summary of the expected number of infections per 100,000 servings for the scenarios is shown in Table 3. Note that it was assumed that all burgers were cooked identically in each scenario; therefore there is no variability between each scenario post processing.

Table 3: Summary of the number of human infections per 100,000 servings for the scenarios considered

Scenario	Burger size	Cooking preference/desired log reduction	Log reduction	Mean number of infections per 100,000
A1	Small	Rare	0.2	28.0
A2	Small	Medium	0.3	19.4
A3	Small	Well-done	0.3	19.4
A4	Small	Well-done + 2	Total inactivation	0*
A5	Small	4 log reduction	4.0	9.9
A6	Small	6 log reduction	6.0	3.3
B1	Standard	Rare	0.5	27.9
B2	Standard	Medium	1.1	25.6
B3	Standard	Well-done	4.1	8.7
B4	Standard	Well-done + 2	Total inactivation	0*
B5	Standard	4 log reduction	4.0	8.7
B6	Standard	6 log reduction	6.0	4.2
C1	Gourmet	Rare	0.8	41.6
C2	Gourmet	Medium	6.5	6.1
C3	Gourmet	Well-done	9	0.9
C4	Gourmet	Well-done + 2	Total inactivation	0*
C5	Gourmet	4 log reduction	4.0	18
C6	Gourmet	6 log reduction	6.0	6.9

*complete inactivation was observed from RIVM thermodynamic model so assume no infections

As expected, the APHA model predicts that consumption of rare burgers results in the highest number of human illness per 100,000 servings compared to medium and well done burgers. This trend is seen for the three sizes considered (small, standard and gourmet). Further, for rare burgers increasing the burger size generally increases the risk of acquiring VTEC O157 infection. This is to be expected given that as the burger sizes increase, the burgers contain more meat and therefore, potentially more VTEC O157. For burgers cooked well done plus an extra 2 minutes, regardless of size, complete inactivation was observed resulting in no human infections.

The model predicts a lower number of infections per 100,000 servings for well-done compared to medium burgers for the gourmet and standard sizes, but not for the small burgers. However, this is likely due to a more complicated relationship between time, temperature and thickness of burger impacting on inactivation of VTEC O157. For small burgers, it takes very little time to get up to the desired temperature for both medium and well done (~1 minute) but this is not sufficient time for a marked reduction in VTEC O157 (i.e. 0.3 log reduction; Table 2). In order to inactivate VTEC O157, not only high temperatures throughout the burger are needed, but also sufficient time for the inactivation process. Accordingly, given the limited inactivation of VTEC O157 within the burger, it is estimated there will be 19.4 human infections per 100,000 servings. This is greater than the 6.1 and 0.9 infections estimated per 100,000 servings of medium and well-done gourmet burgers, respectively. However, for the latter, burgers are cooked between 22 and 36 minutes and can achieve up to 9 log reduction. The number of human infections per 100,000 servings for gourmet burgers achieving a 4 and 6 log reduction was estimated to be 18 and 6.9 respectively, which equates to burgers cooked between rare and medium.

5 Discussion

The discussion focuses on the results from the RIVM thermodynamic model as the APHA risk assessment has been discussed previously (see Berriman *et* al., 2014 for example). Starting with small, 1cm high burgers (scenarios A1 to A6'), the highest temperature is reached rapidly (i.e. within 5 minutes). Shortly after four minutes, the temperature within the burger starts to decrease since then the burger is left to cook at a lower temperature of 100 degrees. The observed log reductions of VTEC O157 for these scenarios are low, around 0.3 logs. This can be understood by noting that the time needed to reach the internal burger temperatures associated with the scenarios is extremely low (i.e. below or just above a minute). The organisms simply do not have enough time to reach a high enough level of inactivation. However, for a 4 or 6-log inactivation, a bit more than two minutes are required, which is long enough to achieve a much greater inactivation.

For the 2.5 cm (or standard) burger, it takes longer for the burger to get thoroughly heated. The scenarios for raw and medium cooking show less than or around one log reduction. After four minutes, i.e. the time required to achieve an internal temperature designated 'medium', the log reduction drastically increases, thus all other scenarios attain at least a 4 log reduction.

The 5 cm high (or gourmet) burger is rather bulky, and it takes a long time for the heat to penetrate to the core. It takes over half an hour (36.7 minutes) for the core to reach well-done temperatures (68.3 ^{o}C). On the other hand, since it takes a lot of time, and the temperatures are well above inactivation temperatures after only a couple of minutes, inactivation is still efficient.

To advise consumers, it is preferable to supply not only a preferred cooking style, but also a minimum time of cooking. The ACMSF cooking recommendations (Table A3.1), for example, are reasonably in line with the results of RIVM thermodynamic model (Table 2). However, for temperatures above 70 degrees, we would still recommend a minimum time of cooking, to give the pathogens time to inactivate. Based on the model assumptions, a minimum of two minutes generates log reductions of greater than 1. However, depending on thickness, rare and medium cooked burgers can be unsafe, giving reductions around or below one log.

The overall conclusions of this report follow a similar theme as that presented within the previous report (Berriman et al., 2014), whereby rare burgers present a higher human health risk compared to those cooked more thoroughly. The results for the well-done and 4/6 log reduction cooking styles were largely in line between the reports, however a considerable difference was seen between rare and medium cooked burgers. Within the original APHA risk assessment, the average log reduction for rare and medium burgers were 1.3 (5th and 95th percentiles of 0.6 and 2.0) and 3.1 (5th and 95th percentiles of 2.4 and 3.8) respectively. As such, it is possible that the original Cassin model (Cassin et al., 1998) overestimates the thermal inactivation for these burger types compared to the more mechanistic RIVM model.

This work has applied the outputs of a more comprehensive cooking model for inactivation of VTEC O157 within a variety of different sized burgers within a previously developed risk assessment model. The models suggest that there is an influential relationship between burger size, internal temperature and cooking duration on the inactivation profile of VTEC O157 within the burger and the subsequent risk to public health. In conclusion, regardless of burger size, consumption of rare burgers results in greater infections with VTEC O157 compared to medium and well-done burgers. Further, cooking a burger so it is well-done (i.e. internal temperature of 68.3ºC) plus an extra 2-minutes results in complete inactivation and therefore no predicted cases of VTEC O157 illness in humans. It needs to be borne in mind, the latter result is based on the model assumptions and inputs and, therefore, the absolute value should be used with caution. There could still be a theoretical risk from consuming such burgers and therefore the risk should not be considered zero but rather negligible.

6 Quality statement

APHA The original model was developed in accordance with the Veterinary Laboratories Agency quality standards at that time (ISO9001). The final report was peer reviewed and commented upon by the FSA. In updating the model (November 2014), it has been internally verified and checked for accuracy (including linked cells, equations) including ensuring the model aligns with the written report and peer-reviewed scientific paper.

RIVM The model was developed under de RIVM/CIb/Z&O quality system. The original model as used in the EFSA risk assessment was extensively reviewed by peers, both internally at RIVM as well as externally. It was published on the EFSA website as an external scientific report. RIVM works according to the ISO9001 quality standard.

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Appendix 1 – Derivation of the thermodynamic model

Background

Diffusion as a function of time and spatial coordinates is governed by the heat equation (see e.g. (Hallström, Skjöldebrand et al. 1988) or (De Jong, Beumer et al. 2005)),

$$\partial_t T(r,\theta,z,t) = \kappa \Delta T(r,\theta,z,t)$$

In a Cylindrical coordinate frame, the Laplacian is defined as

$$\Delta = \frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_{\theta\theta} + \partial_{zz}$$

Let us assume rotational symmetry, implying partial derivatives with respect to θ are zero. The heat equation then becomes

$$\partial_t T = \kappa(\partial_{rr}T + \partial_{zz}T + \frac{1}{r}\partial_r T)$$

The parameter κ [m^2/s] is known as the thermal diffusivity and represents the material properties of the product. It is defined as

$$\kappa = \frac{\lambda}{\rho c_p},$$

with

- ρ, the density of the product [kg/m^3],
- c_p, the specific heat capacity [J/(kg K)],
- λ, the thermal conductivity [W/(mK)].

In (Kumcuoglu, 2009), the thermal diffusivity is both measured and a literature survey is conducted, all sources agree on the value of $\kappa = 1.2 \times 10^{-7}$ m^2/s.

Boundary Conditions

At the boundaries we need boundary conditions. The simplest boundary condition is at the bottom (z=0) where the temperature is kept at the heating temperature T_H,

$$T(r,0,t) = T_H(t).$$

At r=0, the central axis of the burger, we demand that there is no flux in the direction of the boundary. In other words, the temperature gradient ∇T is perpendicular to the unit outward normal n.

$$\nabla T(0,\theta,z,t)\cdot(-1,0,0) = 0 \Rightarrow \partial_r T = 0.$$

At the other boundaries, at r=R and z=H, heat flows depending on the ambient temperature T_A. The description of the boundary condition is based on Newton's law of cooling. This law states that the heat flux is proportional to the temperature difference, and is given by

$$\kappa\partial_n T = -\alpha[T - T_A] \Rightarrow \partial_r T = -\frac{\alpha}{\kappa}[T - T_A]\text{ ,at r=R,}$$

$$\kappa\partial_n T = -\alpha[T - T_A] \Rightarrow \partial_z T = -\frac{\alpha}{\kappa}[T - T_A]\text{, at z=H,}$$

where α is the convective surface heat transfer coefficient in [W/m^2K]. The ambient temperarure T_A is set at 50 degrees Celsius.

Finite Differences

Let us first discretize the function in the *r* and *z* directions, using *N* and *M* grid points in the *r* and *z* directions. Then, the grid spacing's are

$$h_r = W/(N-1).$$
$$h_z = H/(M-1).$$

Also, set $r_i = ih_r$ and $z_j = jh_z$ for $0 \le i < N$ and $0 \le j < M$. The standard discretization resolution is *N=M=20*. Furthermore, we abbreviate $T(r_i, \theta, z_j, t) = T_{ij}$. The central and forward difference approximations to the first derivative (with respect to r) are

$$\partial_r^C T_{ij} = \frac{T_{i+1,j} - T_{i-1,j}}{2h_r},$$
$$\partial_r^F T_{ij} = \frac{T_{i+1,j} - T_{i,j}}{h_r},$$
$$\partial_r^B T_{ij} = \frac{T_{i,j} - T_{i-1,j}}{h_r}$$

The usual 2-point difference approximations to the second derivatives are then

$$\partial_{rr} T_{ij} = \frac{T_{i-1,j} - 2T_{ij} + T_{i+1,j}}{h_r^2}, \text{ for } 1 < i < N-1 \text{ and } 0 \le j < M,$$
$$\partial_{zz} T_{ij} = \frac{T_{i,j-1} - 2T_{ij} + T_{i,j+1}}{h_z^2}, \text{ for } 0 \le i < N \text{ and } 1 < j < M-1.$$

Combining these with the central difference for the first derivative in the r-direction, we find the 5-point difference method for the heat equation on the interior,

$$\partial_t T_{ij} = \xi_W T_{i-1,j} + \xi_E T_{i+1,j} + \xi_C T_{ij} + \xi_N T_{i,j+1} + \xi_S T_{i,j-1},$$
$$\xi_W = \frac{\kappa}{2ih_r^2}(2i-1),$$
$$\xi_E = \frac{\kappa}{2ih_r^2}(2i+1),$$
$$\xi_N = \frac{\kappa}{h_z^2}, \xi_S = \frac{\kappa}{h_z^2},$$
$$\xi_C = -2\kappa\left(\frac{1}{h_z^2} + \frac{1}{h_r^2}\right).$$
,

for $1 < i < N-1$ and $1 < j < M-1$. Note that at points close to the boundary we need unknown values, e.g. $T_{0,j}$ is needed for calculation of $\partial_t T_{1j}$. This values are known from the boundary conditions, discretised using forward differences (FD) or backward diffences (BD),

$$T_{i,0} = T_H,$$
$$T_{1,j} = T_{0,j}, (FD)$$
$$T_{N-1,j} = \alpha_1 T_{N-2,j} + \alpha_2, (BD),$$
$$T_{i,M-1} = \alpha_1 T_{i,M-2} + \alpha_2, (BD),$$
$$\alpha_{1,r} = \frac{\kappa}{\kappa + h_r \alpha}, \alpha_{2,r} = \frac{h_r \alpha}{\kappa + h_r \alpha} T_A.$$

The following table lists the corrrections to be applied near the boundary,

	Boundary	ξ_C correction	r correction
j=1	$\xi_S = 0$		$\xi_S T_H$
j=M-2	$\xi_N = 0$	$\alpha_1 \xi_N$	$\alpha_2 \xi_N$
i=1	$\xi_W = 0$	ξ_W	
i=N-2	$\xi_E = 0$	$\alpha_1 \xi_E$	$\alpha_2 \xi_E$

Let is now number the grid points using, $\varphi(i,j) = (j-1)(N-2) + i$ and define the vector x as $x_{\varphi(i,j)} = T_{ij}$. Now we can turn the above equations into a matrix-vector system, if we put the equations in the rows of *A*, indexing by $\varphi(i,j)$, and the coefficients in the columns of *A*, for *1<i<N-2* and *1<j<M-2*

$$A_{\varphi(i,j),\varphi(i,j+1)} = \xi_N, A_{\varphi(i,j),\varphi(i,j-1)} = \xi_S, \text{etc.}$$

Then

$$x'(t) = Ax(t) + r$$

Having solution

$$x(t) = e^{At}(x(0) + A^{-1}r) - A^{-1}r$$

perform the singular value decomposition $A = V\Lambda V^{-1}$, then

$$\exp(A) = V \exp(\Lambda) V^{-1} = V \begin{bmatrix} \exp(\lambda_1) & 0 & 0 \\ 0 & \ldots & 0 \\ 0 & 0 & \exp(\lambda_n) \end{bmatrix} V^{-1}$$

and $A^{-1} = V\Lambda^{-1}V^{-1}$. With the substitution $q = \Sigma^{-1}V^{-1}s$ and $x = Vy$ the solution may be written

$$y(t) = \exp(\Sigma t)(y_0 + q) - q$$

This enables us to efficiently calculate the solution for any point in time.

Divide n VTEC over the cells.

When dividing n VTEC over a cylinder of height H, and half width W, the concentration is,

$$C = \frac{n}{\pi W^2 H}.$$

In a cell *(i,j)*, the amount of VTEC actually correspond to the VTEC in the entire annulus associated to that cell. There are *NxM* grid points, corresponding to *(N-1)x(M-1)* cells. The coordinate of the cell at index *(i,j)* is

$$(i\frac{W}{N-1}, j\frac{H}{M-1}).$$

Thus, the number of VTEC in the ring defined by this cell is

$$S(i,j) = \int_{jH/(M-1)}^{(j+1)H/(M-1)} \int_{iW/(N-1)}^{(i+1)W/(N-1)} \int_0^{2\pi} Cr \quad d\theta \quad dr \quad dz = \frac{2\pi CH}{M-1}\frac{1}{2}\left[r^2\right]_{iW/(N-1)}^{(i+1)W/(N-1)}$$
$$= \frac{(2i+1)n}{(M-1)(N-1)^2}$$

As a check we can sum this over all grid cells,

$$\sum_{j=0}^{M-2}\sum_{i=0}^{N-2} S(i,j) = \frac{n}{(M-1)(N-1)^2}\sum_{j=0}^{M-2}\sum_{i=0}^{N-2}(2i+1) = \frac{n}{(N-1)^2}(N-1+2\sum_{i=0}^{N-2} i)$$
$$= \frac{n}{(N-1)^2}(N-1+2\sum_{i=0}^{N-2} i)$$
$$= \frac{n}{(N-1)^2}(N-1+2\sum_{i=1}^{N-1}(i-1)) = \frac{n}{(N-1)^2}\{N-1+2([\sum_{i=1}^{N-1} i]-(N-1)\} =$$
$$\frac{n}{(N-1)^2}\{N-1+2(\frac{(N-1)^2+N-1}{2}-(N-1)\} = n$$

The *S(i,j)* may be used to divide the VTEC over the burger. At each time step in the simulation, the total number of VTEC is determined by summing over the cells.

Appendix 2 - ACMSF cooking times

The ACMSF cooking times for burgers which inform the FSA guidelines of cooking at 70^{o}C for 2 minutes are summarized in Table A3.1.

Table A3.1. ACMSF cooking recommendations[i]

Temperature [°C]	Time
60	45 min
65	10 min
70	2 min
75	30 s
80	6 s

This table concerns core temperature and time, and is thought to be sufficient for a six log reduction.

[i] ACMSF (1998). Report on the safe cooking of burgers. Available http://www.food.gov.uk/sites/default/files/multimedia/pdfs/acmsfburgers0807.pdf